NKG2016LU, an improved postglacial land uplift model over the Nordic-Baltic region

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NKG2016LU is a semi-empirical land uplift model computed in Nordic-Baltic cooperation in the NKG Working Group of Geoid and Height Systems.

The model gives the vertical land uplift rate in two different ways,

- **NKG2016LU_abs**: Absolute land uplift in ITRF2008 (i.e. relative to the Earth’s centre of mass)
- **NKG2016LU_lev**: Levelled land uplift, i.e. uplift relative to the geoid.

No apparent model (i.e. uplift relative to Mean Sea Level over a certain time period) is released for the time being.

- Due to the (accelerating) contemporary climate-related sea level rise (caused by temperature increase, present day ice melting, etc.), the apparent land uplift is not equal to the levelled land uplift.

NKG2016LU has been computed based on

- An empirical land uplift model computed by Olav Vestøl based on geodetic observations (GNSS time series from BIFROST and NKG levelling, no tide gauges used)
- The preliminary geophysical GIA model **NKG2016GIA_prel0306** computed by Steffen et al. (2016) in the NKG WG of Geodynamics.
Introduction

- **An empirical land uplift model** is computed directly from the observations using a mathematical method, like for instance least squares collocation. (In Ågren and Svensson, 2007, this type of model is called “mathematical model”)

- A **geophysical GIA model** is computed in a geophysically meaningful way based on an Earth model, an ice melting history, etc. (GIA=Glacial Isostatic Adjustment).

- A **semi-empirical land uplift model** is a combination of an empirical model and a geophysical GIA model.

- The previous official semi-empirical postglacial land uplift model **NKG2005LU** was originally computed for the adjustment of the Baltic Levelling Ring (Vestøl 2007; Ågren et al. 2007).

- NKG2005LU is based on an empirical model computed from GNSS, levelling and tide gauges, which was then combined with the geophysical GIA model of Lambeck et al. (1998b) as is described in Ågren and Svensson (2007).

- In 2011, the NKG WG of Geoid and Height System started a new project to compute an improved version of NKG2005LU with Olav Vestøl as project leader. **NKG2016LU is the final result of this project**.

- In 2013, NKG under the leadership of Holger Steffen started to develop and compute GIA models. This activity, which involves more or less all GIA-modellers in the Nordic-Baltic countries, has an active cooperation with Lev Tarasov regarding the construction and tuning of ice models.

The preliminary version **NKG2016GIA_prel0306** (Steffen et al. 2016) is used for NKG2016LU.
The (strictly) empirical model
Basic concepts for the empirical model

• **Geodetic observations** alone are used to calculate the **absolute** land uplift in ITRF2008. The following observations are used:
  
  – GNSS (vertical) velocities in CORS, from the BIFROST 2015/16 calculation processed in GAMIT/GLOBK. Finalised in March 1, 2016; an updated version of Kierulf et al. (2014).
  
  – Levelling from all the Nordic countries (except Iceland) and from all the Baltic countries.

• **Least squares collocation with unknown parameters** to estimate the absolute uplift in the observation points. (Separate gridding algorithm utilised by Vestøl, but this one is not utilised for NKG2016LU)

• **Trend surface** consisting of a 5th degree polynomial. Least squares collocation to estimate an additional **signal** (=difference from trend surface). A first order Gauss Markov covariance function with halved correlation after 40 km and variance (0.3 mm/year)$^2$ is selected for this latter part of the solution.

• The **geoid rise** is needed to relate the levelling and GNSS observations. This quantity is now taken directly from the GIA model (see below).
  
  – This means that the empirical model is actually not strictly empirical (but almost!)
  
  – However, almost the same empirical absolute land uplift values are obtained when solving for a scale factor to describe the geoid rise, (below ~0.1 mm/year everywhere).
  
  – This means that in practice the empirical model can be regarded as a strictly empirical model.
The levelling network

Levelling data used for the empirical model behind NKG2005LU

New data included since 2005

- 1 time
- 2 times
- 3 times
- ≥4 times
The CORS

Stations 2005

New dataset


BIFROST 2015/16 calculation processed in GAMIT/GLOBK. Finalised March 1, 2016; an updated version of Kierulf et al. (2014).
The estimated signal and model

Signal estimated by least squares collocation
(in the observation points, then gridded)

Purely empirical model (polynomial + signal)
(absolute uplift in the observation points,
then gridded)
GNSS rate residuals (difference between the BIFROST solution and the gridded empirical model)

The removed observations are not shown.

<table>
<thead>
<tr>
<th>Station</th>
<th>Residual</th>
<th>Outlier</th>
<th>T-value</th>
</tr>
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<tr>
<td>Removed</td>
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<tr>
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<td></td>
<td>&lt;2</td>
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<tr>
<td>VAAS</td>
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<td>-1.0</td>
<td>2.4</td>
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</table>
Levelling data - Some results

<table>
<thead>
<tr>
<th>Est. Accuracy (mm/km)</th>
<th># outliers removed</th>
<th>Remark</th>
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<tbody>
<tr>
<td>1. 2. 3.</td>
<td>1. 2. 3.</td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>2.4 1.4 1.3</td>
<td>2 6 7</td>
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<tr>
<td></td>
<td></td>
<td>1. is railway levelling.</td>
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<tr>
<td>Finland</td>
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<td>1 0 1</td>
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<tr>
<td>Sweden</td>
<td>3.0 1.4 1.1</td>
<td>0 7 2</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.4 0.9</td>
<td>0 0</td>
</tr>
<tr>
<td>Estonia</td>
<td>0.9 0.4</td>
<td>21 1 *</td>
</tr>
<tr>
<td>Latvia</td>
<td>0.7 1.0</td>
<td>0 0</td>
</tr>
<tr>
<td>Lithuania</td>
<td>0.6 1.4</td>
<td>1 1</td>
</tr>
</tbody>
</table>

* Many obs. in 1. levelling removed due to sinking problems in Parnu.
The underlying GIA model
Method overview

- Viscoelastic normal-mode method, pseudo-spectral approach to solve the sea level equation. Applying software ICEAGE
- Spherically symmetric (1D), compressible, Maxwell-viscoelastic earth model
- Lithospheric thickness, upper and lower mantle viscosity as free parameter (so-called three-layer earth model)

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithospheric thickness [km]</td>
<td>60, 80, 90, 100, 120, 140, 160</td>
</tr>
<tr>
<td>Upper mantle viscosity [10^{20} Pa s]</td>
<td>1, 2, 4, 7, 10, 20, 40</td>
</tr>
<tr>
<td>Lower mantle viscosity [10^{22} Pa s]</td>
<td>0.1, 0.2, 0.4, 0.7, 1, 2, 4, 7, 10</td>
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</tbody>
</table>

- 25 individual ice history models was tested
- The combination of values in the earth model and the 25 ice history models gives more than 11 000 GIA models (earth-ice model combinations)
- All these 11 000 GIA models are tested in a root-mean-square fitting to observations.
  - GNSS uplift rates
  - RSL data

\[ \chi = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{o_i - GIA_i(a_j)}{\Delta o_i} \right)^2} \]
RSL (Relative Sea Level) observations

- Any geological marker giving information about the past position of the sea level
- Data from Lambeck et al. 1998a, Vink et al. 2007 and Steffen et al. 2014
- In total 700 RSL-observations
- Different types:
  - Drowned former land surfaces
  - Uplifted shorelines and beaches
  - Isolation basins

RSL observations in this connection are of the ancient sea level relative to land… and must not be mixed up with modern tide gauge records.
Difference between the BIFROST GNSS and NKG2016GIA_prel0306

- mm/year
- Statistics:

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<tbody>
<tr>
<td>#</td>
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<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>Max</td>
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<td></td>
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<tr>
<td>Mean</td>
<td>-0.06</td>
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<tr>
<td>StdDev</td>
<td>0.61</td>
<td></td>
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</table>
NKG2016GIA_prel0306 – vertical land uplift

- Gridded vertical displacement rate
- Statistics:
  - # 313 x 301
  - Min -4.55
  - Max 10.49
  - Mean 0.90
  - StdDev 3.22
- Contour interval: 0.5 mm/year
NKG2016GIA_prel0306 – Geoid rise

- Gridded geoid change rate
- Statistics:
  - #: 313 x 301
  - Min: -0.05
  - Max: 0.66
  - Mean: 0.21
  - StdDev: 0.28
- Contour interval: 0.1 mm/year
Computation of the final models NKG2016LU_abs and NKG2016LU_lev
Computation of NKG2016LU_abs and NKG2016LU_lev

• The empirical estimates in the observation points and the estimated standard errors are then combined with the GIA model NKG2016GIA_prel0306 using the following remove-compute-restore technique:
  – The GIA model is first removed from the empirical model in the observation points
  – Least Squares Collocation (LSC) is applied to model the differences from the GIA model (residual surface). A first order Gauss Markov covariance function with correlation length 150 km used (chosen based on covariance analysis). The estimated standard errors above applied for the observations.
  – The residual surface grid is finally restored to the GIA model to obtain the final land uplift grid NKG2016LU_abs,

\[
\hat{h}_{\text{grid}}^{\text{NKG2016LU_abs}} = \hat{h}_{\text{grid}}^{\text{NKG2016GIA_prel0306}} + \text{LSC}\left\{\hat{h}_{\text{obs. points empirical_abs}}^{\text{NKG2016GIA_prel0306}} - \hat{h}_{\text{obs. points NKG2016GIA_prel0306}}\right\}
\]

• The levelled uplift (relative to the geoid) is then computed by subtracting the GIA model geoid rise according to

\[
\hat{H}_{\text{grid}}^{\text{NKG2016LU_lev}} = \hat{h}_{\text{grid}}^{\text{NKG2016LU_abs}} - \hat{N}_{\text{grid}}^{\text{NKG2016GIA_prel0306}}
\]
Difference between the empirical model in the observation points and the GIA model

\[ \hat{h}_{\text{obs. points}} - \hat{h}_{\text{empirical abs}} = \frac{\hat{h}_{\text{obs. points}}}{\text{GIA model}} \]

- mm/year
- Statistics:
  - # 1111
  - Min -1.23
  - Max 1.24
  - Mean -0.13
  - StdDev 0.34
Residual surface
(gridded difference between the empirical model in the observation points and the GIA model)

\[\text{Residual surface (grid)} = \text{LSC}\left\{ \hat{h}_{\text{obs. points}} - \hat{h}_{\text{empirical_abs NKG2016GIA_prel0306}} \right\}\]

- Interpolation method: Least Squares Collocation (LSC). A first order Gauss Markov covariance function with correlation length 150 km used (chosen based on covariance analysis). The estimated standard errors above applied for the observations.

- Statistics:

  - # 313 x 301
  - Min -1.14
  - Max 0.88
  - Mean 0.00
  - StdDev 0.27

- Contour interval: 0.1 mm/year
Absolute land uplift in ITRF2008 (relative to the centre of mass)

Statistics:

- # 313 x 301
- Min -4.61
- Max 10.29
- Mean 0.90
- StdDev 3.14

Contour interval: 0.5 mm/year

Should be used for the correction of GNSS or other space geodetic techniques.
Residuals (i.e. difference between the empirical model in the observation points and NKG2016LU_abs)

- mm/year
- Statistics:

  - #: 1111
  - Min: -0.15
  - Max: 0.45
  - Mean: 0.00
  - StdDev: 0.04
Levelled uplift = uplift relative to the geoid.

The geoid is here interpreted as an equipotential surface that is still rising due to historical ice melting in the past, through Glacial Isostatic Adjustment..., but **not** due to contemporary climate related sea level changes (caused by temperature increase, present day ice melting, etc.)

Statistics:
- #: 313 x 301
- Min: -4.67
- Max: 9.63
- Mean: 0.69
- StdDev: 2.98

Contour interval: 0.5 mm/year

Can be used for epoch conversion of orthometric or normal heights in a vertical reference system.

Can also be utilised as a basis to take care of the postglacial land uplift due to old historic deglaciations in sea level studies; see next paragraph.
The uncertainty of the final combined model

- Method
  1. Replace the polynomial trend with the GIA-model
  2. Estimates the differences to the GIA-model as signals
     - Based on the differences between the empirical model and the GIA-model we find empirically the covariance of these signals.
  3. Recalculate the empirical model
Why not use tide gauges in the computation of the absolute uplift of the empirical model?

Difference of gridded empirical model computed with and without tide gauges (meters):

- Since there may be spatial variations in the mean sea level rise, it is difficult to separate this effect from the land uplift.
- There is also temporal variations in the mean sea level rise, making the separation even more difficult. The apparent uplift computed in the tide gauges will always refer to a certain time interval.
- Another advantage is that the final model NKG2016LU_lev then becomes independent from tide gauge and sea level related information.
- We can then learn something about climate related sea level changes by comparing NKG2016LU_lev with apparent uplift in tide gauges for a certain time period.
- The differences are overall small and almost negligible, except the northernmost part of Norway. (See figure to the left).
Comparison with the old model (NKG2005LU)
Differences between NKG2016LU_abs and NKG2005LU_abs

• Old official model; cf. introduction (Vestøl 2007; Ågren et al. 2007).

• Different reference frames ITRF2008 vs ITRF2000)

• Statistics (mm/year):

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<thead>
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<tbody>
<tr>
<td>#</td>
<td>313 x 301</td>
</tr>
<tr>
<td>Min</td>
<td>-3.89</td>
</tr>
<tr>
<td>Max</td>
<td>3.10</td>
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<tr>
<td>Mean</td>
<td>-0.25</td>
</tr>
<tr>
<td>StdDev</td>
<td>1.35</td>
</tr>
</tbody>
</table>

• Contour interval: 0.05 mm/year

• The strange “ridge” e.g. outside the west coast of Norway depends on that NKG2015LU was deliberately truncated to the apparent uplift -2 mm/year (smaller values set to this)

• The big deviation in the White Sea is due to different ice models.
Final words

• The semi-empirical land uplift model **NKG2016LU** is hereby released.

• The model gives the vertical land uplift rate in two different ways,
  – **NKG2016LU_abs**: Absolute land uplift in ITRF2008 (i.e. relative to the Earth’s centre of mass)
  – **NKG2016LU_lev**: Levelled land uplift, i.e. uplift relative to the geoid.

• The NKG2016LU_abs and NKG2016LU_lev have been computed based on
  – An **empirical land uplift model** computed by Olav Vestøl (2016) based on geodetic observations (GNSS time series from BIFROST and NKG levelling, *no tide gauges used*)
  – The preliminary geophysical GIA model **NKG2016GIA_prel0306** computed by Steffen et al. (2016) in the NKG WG of Geodynamics
  – The geoid rise of this GIA model is used to transform between absolute and levelled uplift.

• No apparent model (i.e. uplift relative to Mean Sea Level over a certain time period) is released for the time being,
  – Due to the (accelerating) contemporary climate-related sea level rise (caused by temperature increase, present day ice melting, etc.), the apparent land uplift is **not** equal to the levelled land uplift.
  – If the **apparent uplift** is needed, then it is recommended to estimate a constant (for a certain time interval and for a certain geographical area) to subtract from NKG2016LU_lev. This is a qualified task that should be made with great care.