Acoustic camera and beampattern

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***Abstract*—The wavenumber-frequency response of an array describes the response to an arbitrary plane wave both in time and space. When the input consists of a single monochromatic plane wave, the response of the array is referred to as the array pattern or beampattern. Through the design of the array our goal is to have a directivity of the array as high as possible in a given direction, while suppressing signals and noise from directions being different from our focused attention. The beampattern of an array is a key element in determining array performance, and is largely determined by the array geometry.**

***Index Terms*—Array pattern, beampattern, beamforming, wavenumber-frequency space**

INTRODUCTION

AN acoustic array consists of a number of elements, sensors or microphones positioned in a certain geometry that are reacting to sound wave fields hitting the array to produce outputs. When we observe a wavefield through an array of finite size, the observed wavefield spectrum output from the array will be a convolution between the source spectrum and the array, and becomes a distorted and smoothed version of the source spectrum. Our goal is to design an array such that the observed spectrum is as close to the true spectrum as possible. We can control the impact the array has on the spectrum by changing the size and shape of the array, the number of elements being used and assigning different weights to different elements of the array. The weighting is sometimes also referred to as shading, tapering or apodization. In general an array with a large spatial extent can be more focused on any specific direction.

A single microphone element of an array will sample the wavefield in time, and we can then get the well known frequency response of time-varying signals. Since an array consists of several sensors located at different positions, the total array not only samples the wavefield in time, but also in space based on the position of the individual elements. Where the transformation of time will be frequency response, the transformation of space is given as the wavenumber response, and the total array response to an arbitrary wave corresponds to the wavenumber-frequency response of a filter operating both in time and space (spatio-temporal). That is, the wavenumber-frequency response is to an array what the frequency response is to a digital filter.

When the input to the array is a single monochromatic (single frequency) plane wave in a homogenous medium, the total wavenumber-frequency response of the array is called the *array pattern*, or *beampattern*. The beampattern is the array response plot when the array is steered to a direction of interest, and evaluates the magnitude of the array output as a function of the incoming angle for the sound waves. Through the beampattern we can analyse how the array's output is disturbed by signals being different from the one of our focused attention. The goal in array signal processing is to combine the different elements of the array in such a way that we are able to steer the focus or beam to our desired direction. This means that signals at particular angles experience constructive interference, while others experience destructive interference.

Because a superposition of plane waves expresses an arbitrary wavefield, it suffices to determine the arrays response to a monochromatic plane wave of a certain frequency and propagation direction in order to provide a complete characterization of the system's frequency response. Or said in other words, to find the beampattern, we use a plane wave as input, and measure the output of the array.

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I. BEAMPATTERN OF ONE-DIMENSIONAL ARRAYS

One of the simplest ways of interpreting beampattern is by assuming a 1D array along the x-axis, measure its beampattern only on the xy-plane, and plotting the beampattern at the angle perpendicular to the line or the plane of the array, also known as the broadside steering angle. An example beampattern for a 10 element 1D array with half wavelength spacing between elements is given in Fig. 1. The beampattern in the figure may be plotted as a function of wavenumber or incidence angle, and plotted in both rectangular plot and polar plot. Although the plots are different, they all convey the same information. Seen in the beampattern is the main beam called the main lobe corresponding to the steering direction, and smaller secondary lobes, called side lobes, which do not correspond to the direction in which the array is being steered. The width of the main lobe determines the array resolution, or the ability to separate sources. The strength of the side lobes compared to the main lobe determine the dynamic range, or contrast. It measures how much attenuation is provided outside the steering direction, and indicates the sensitivity of the array to waves coming from other angles than our steering direction. Our goal is to make the main lobe as narrow and directive as possible, while at the same time suppressing the magnitude of the side lobes. When optimizing an array with different weightings of the elements, the main lobe width and side lobe level will usually be at odds with each other. Lowering the side lobes will lead to an increase in the main lobe width, and finding an optimal beampattern will always involve a compromise between the two. In Fig. 2 is the beampattern for the same array no longer plotted at the broadside steering angle, but rather at 20°.

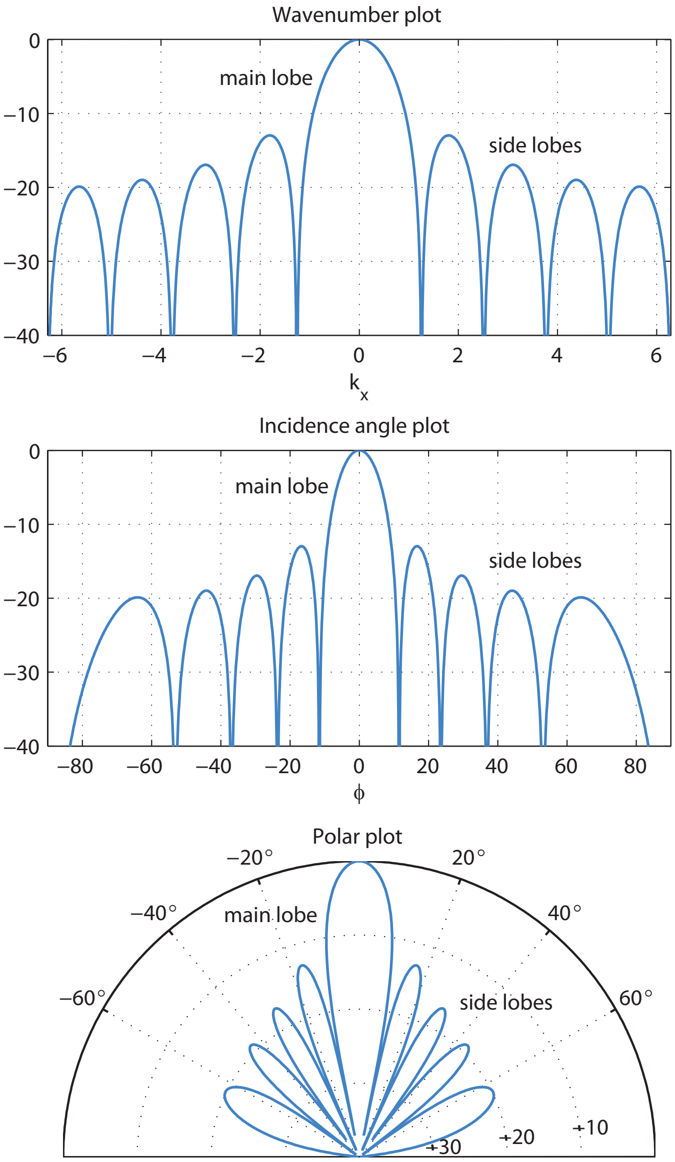

Fig. 1. Beampattern in both rectangular wavenumber plot, rectangular incidence angle plot and polar plot at broadside steering angle

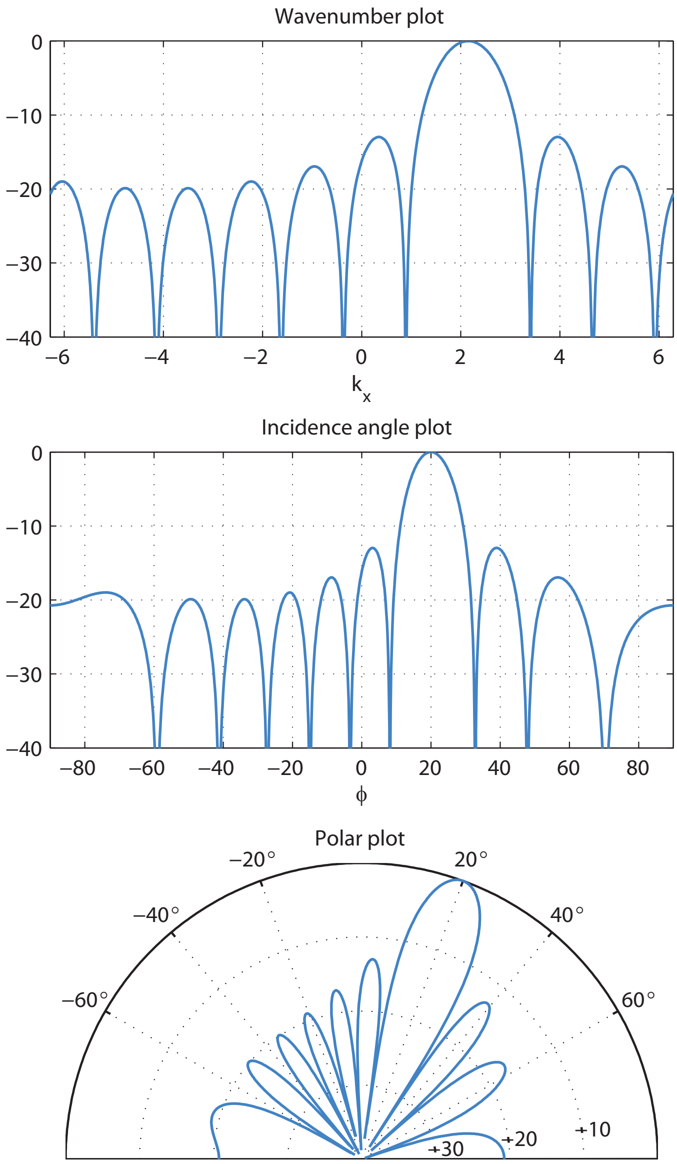

Fig. 2. Beampattern in both rectangular wavenumber plot, rectangular incidence angle plot and polar plot at steering angle 20°

A. Sampling and element distance

As a wavefield hits an array consisting of several sensors, the wavefield is sampled by each individual element and combined. Seen in Fig. 3 is the beampattern of a linear array with M = 10 elements and different element spacings. The elements are uniformly spaced, with three different element spacings: $d = \lambda/2$, λ and 2λ. As the inter-element distance increases, the mainlobe gets narrower in addition to more and narrower sidelobes. However, as the inter-element spacing passes the limit $d \geq \lambda$, so called grating lobes start to appear in the beampattern. Grating lobes have the same magnitude and shape as the main lobe, but occur in other directions than our focus direction. The grating lobes are undesirable, since the array is as sensitive to waves coming from the directions of the grating lobes as for the steering direction. This will severely impair the obtained signal quality.

Grating lobes is a manifestation of inadequate spatial sampling for an array with uniformely spaced element distances, and may be avoided by requiring that the distance between elements is smaller than half the wavelength of the received signal, $d < \lambda/2$. Realising that audible sound spans the frequency range from 20 Hz to 20 kHz, giving a wavelength λ spanning from 17 meters for the lowest frequencies, to 1.7 cm for the highest frequencies, the inter-element distance of the sensors of an ideal array would have to vary from 0.85 centimeters up to 8.5 meters in order to avoid grating lobes for all audible sound.

A different solution is what is known as irregular arrays, where the distance between elements is non-periodic and varies. For such arrays a low side lobe level may be achieved over a wide selection of frequencies even though the average element spacing is much larger than half a wavelength.

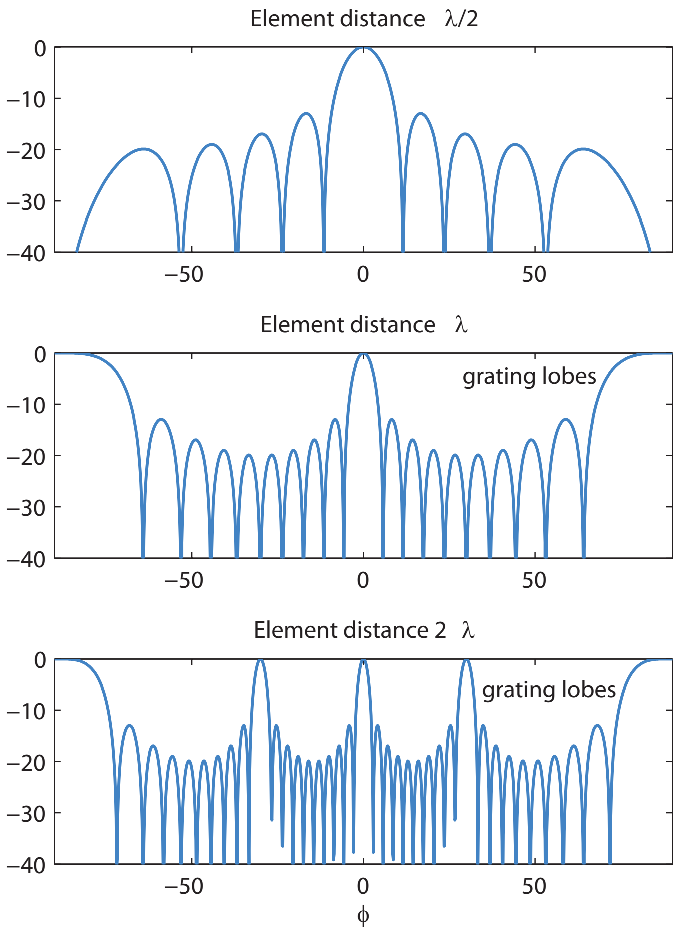

Fig. 3. Beampattern and grating lobes when varying element distance

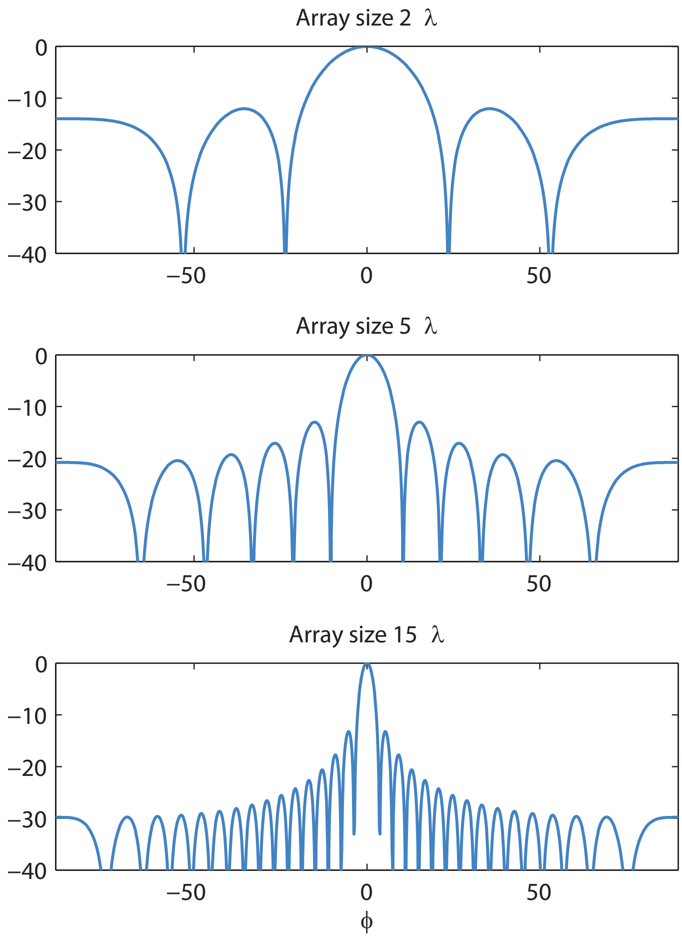

Fig. 4. Beampattern with increasing array size with $\lambda/2$ element spacing

B. Altering array size

Seen in Fig. 4 is the beampattern for three different array sizes, all with inter-element distance of $\lambda/2$ and uniform weighting. As the array size increases, so does the number of elements. As can be seen the relative size of the array greatly affects the sharpness of the beampattern. For arrays with uniform weight and uniform spacing between array elements, as the size increase, the highest side lobe level becomes independent of the number of elements and will be around -13 dB lower than the main lobe. The main lobe beamwidth however will get narrower with increasing size. In order to achieve even lower side lobe levels, one must use non-uniform weights on the elements.

C. Altering element weights

Seen in Fig. 5 is the element weight function and beampattern of an $M = 10$ element array with inter-element spacing of $\lambda/2$. Now the sensor weighting is no longer equal for alle elements, and each sensor will contribute differently to the combined output signal. The gray line in the lower part of the figure represents an array with equal number of elements and inter-element spacing, but with uniform weighting on each sensor. As can be seen the side lobe level is lowered significantly, although at a cost of slightly larger main lobe.

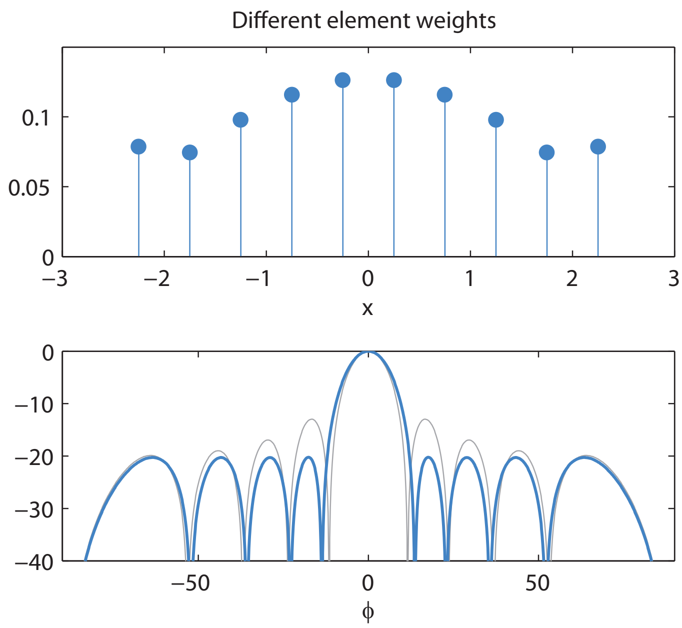

Fig. 5. Beampattern of 10 element array when altering the element weights. The gray line shows the case for uniformly weighted elements

D. Altering element position

Fig. 6 shows the sensor distance and beampattern of an $M = 10$ element array with uniform weighting. The inter-

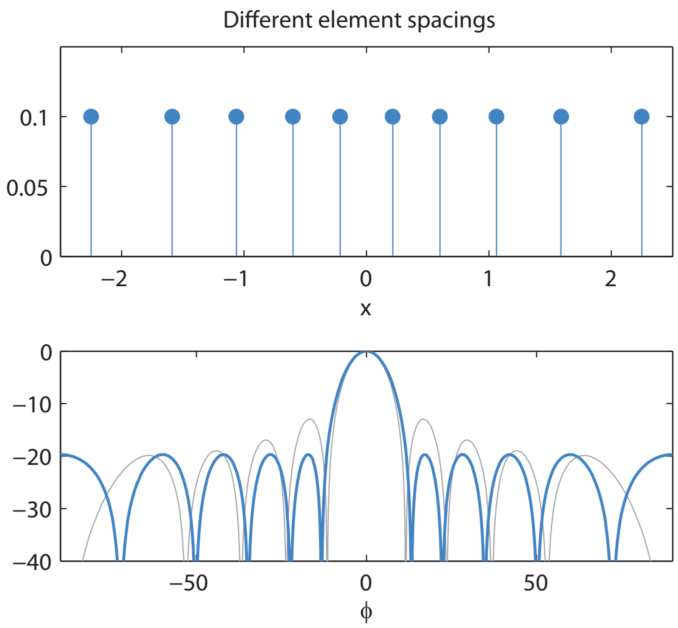

Fig. 6. Beampattern of 10 element array when altering the inter-element spacing. The gray line shows the case for uniformly spaced elements

element spacing however is no longer set at a fixed value, but varies between elements. The distances are symmetric around the centre of the array. Comparing again with the gray line representing the uniformly spaced and uniformly weighted array, one can see how the side lobes are lowered. Once again this comes at a cost of a slightly wider main lobe.

II. Beampattern of two-dimensional arrays

As opposed to the 1D case, the beampattern of a 2D array is plotted in two-dimensional wavenumber space. Shown in Fig. 7 and Fig. 8 is the geometry and beampattern of a rectangular and circular array, both containing the same number of elements with uniform weight. The sides of the rectangular array is 1 meter in extent, whereas the circular array has a diameter of 1 m.

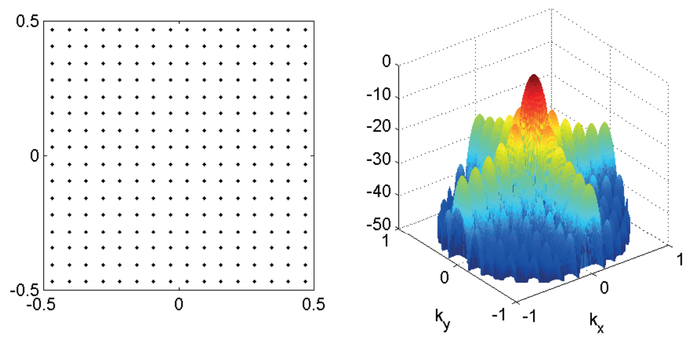

Fig. 7. Geometry and beampattern of 256 element rectangular array at f = 2 kHz

As seen from the beampatterns, just by altering the array geometry from rectangular to circular the side lobe levels are greatly reduced. High side lobe levels and grating lobes give rise to the so called ghost-spot or ghost image effect -

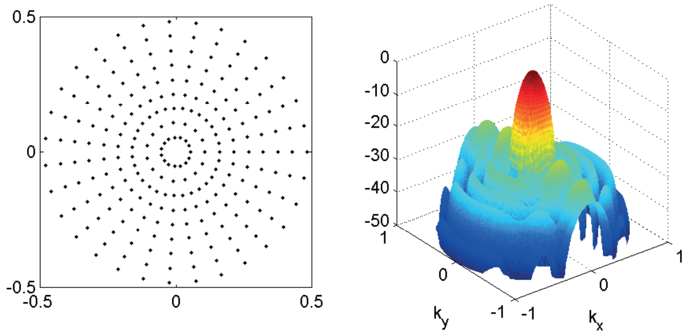

Fig. 8. Geometry and beampattern of 256 element circular array at f = 2 kHz

you measure a source which does not exist. The side lobes may also receive interfering signals from different directions than our focused attention, and increase the noise level in the receiver. The ability to suppress these ghost images is given by the level of the side lobes relative to the mainlobe. Minimizing side lobe levels is crucial for the performance of the array.

III. Norsonic NOR848A acoustic camera

Fig. 9. Geometry of Norsonic NOR848 acoustic camera

The Norsonic NOR848A sets a new standard for acoustical cameras. The large number of microphones eliminates the problems of ghost-spots, compared to an acoustic camera with low number of microphones and high side lobe effect. Seen in Fig. 10 is the array geometry and beampattern at f = 2 kHz for the NOR848A 1.0 m array with 256 microphones.

By having non-uniform element distance the camera avoids problems related to grating lobes. Through clever positioning of the array elements, along with element weighting changing according to the input frequency, the side lobe levels are greatly reduced, along with a sharpened mainlobe compared to the beampatterns of the uniformely weighted rectangular and circular array.

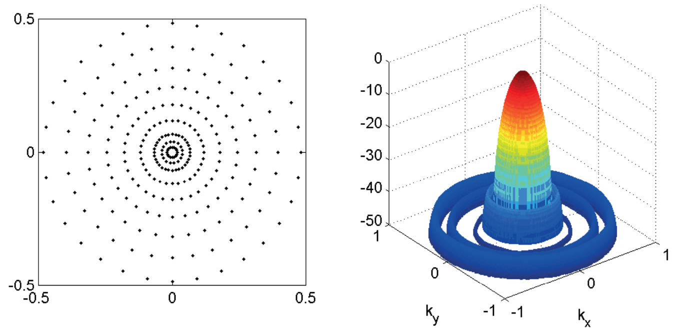

Fig. 10. Geometry and beampattern of 256 element Norsonic NOR848A-10 acoustic camera at f = 2 kHz

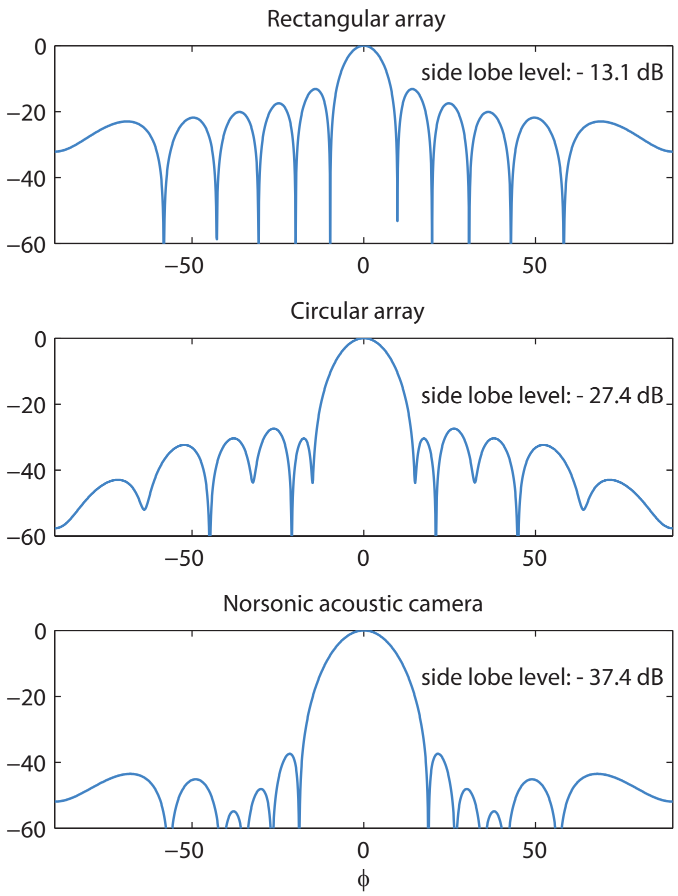

Fig. 11. Beampattern for rectangular and circular array, and Norsonic NOR848A-10 acoustic camera at f = 2 kHz. All arrays have the same number of elements

As seen from Fig. 10 the NOR848A acoustic camera exhibits a circular symmetry in the beampattern with evenly distributed spatial response. The mainlobe is narrower for better zoom and low side lobes ensures no ghost signals. The distribution of the high number of microphones ensures high resolving power and reduces the problems due to side lobe effects, while the digital microphones ensure large dynamic range and high stability. Comparing the beampatterns at frequency f = 2 kHz for the three different array geometries as seen in Fig. 11, the Norsonic NOR848A has a side lobe level -10 dB lower than a circular array, and -24 dB lower than a rectangular array for the same number of elements.

APPENDIX

The *wavenumber vector* (or wave vector) of a plane wave $\vec{k} = \{k_x, k_y, k_z\}$ is the propagation vector giving both the magnitude and direction of arrival of the incident plane wave. Assuming a plane wave with wavelength λ, the magnitude of the wavenumber vector is the *wavenumber* of the wave $|\vec{k}| = k$ measured in units of radians per meter.

$$|\vec{k}| = k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda} \quad (1)$$

where f is the frequency of the incident plane wave (ranging from 20 Hz to 20 kHz for audible sound) and c is the speed of sound waves in air approximately 340 m/s.

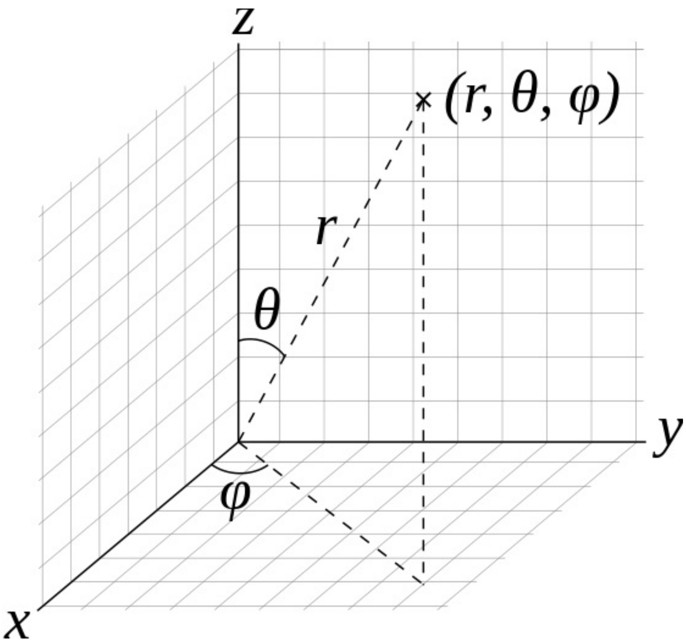

Fig. 12. Spherical coordinate system (image courtesy of Wikipedia)

In most situations, a three-dimensional Cartesian grid represents space, with time being the fourth dimension. Other coordinate systems may be defined as well, and for certain problems, it's more convenient to use spherical coordinates. Here a point is represented by its distance r from the origin, its azimuth ϕ within the equatiorial plane, and its angle θ down from the vertical axis. In Fig. 12 a right handed orthogonal coordinate system is depicted along with a spherical coordinate system. The angle θ is known as the elevation and is the normal incidence angle, and ϕ is denoted the azimuth which is the angle in the XY plane. For a wave propagating in spherical coordinates, the wave vector is related to the Cartesian coordinates by simple trigonometric formulas

$$\begin{aligned} k_x &= k \sin\theta \cos\phi \\ k_y &= k \sin\theta \sin\phi \\ k_z &= k \cos\theta \end{aligned} \quad (2)$$

where the x-component of the wave vector, k_x, determines the rate of change of the phase of a propagating plane wave in the x-direction. The same definitions apply for the y- and z-directions.

When describing the response of an array which is not discrete, but can be sampled at all points within an area, the term *aperture smoothing function* is used. The aperture smoothing function of an array is given as

$$W(\vec{k}) = \int_{-\infty}^{\infty} w(\vec{x}) e^{j\vec{k}\vec{x}} \quad (3)$$

where $\vec{x}$ describes the extent of the array. The discrete version of the aperture smoothing function is named the *array factor* (AF), and describes the spatial response of an M element array. The total response of an array is called the *array pattern*, or *beampattern* (BP), and will not only be a function of the array geometry, but also of the radiation pattern of the individual elements. The spatial response of the element is called the *element pattern* (EP). The beampattern which gives the complete pattern representation of the array can be found by multiplying the array factor and the element pattern. This assumes that the element pattern is identical for each element. The beampattern of an array can then be stated as

$$BP = EP \cdot AF \quad (4)$$

For an array with isotropic elements, the beampattern of the array is the same as the array factor. The positions of the M array sensors, or array elements, are given as

$$\vec{x} = \{x_m, y_m, z_m\} \quad (5)$$

where m ranges from 0 to $M-1$ and each sensor has a weight w_m. The total beampattern of an array with isotropic elements is the weighted sum of the individual elements and may be calculated as

$$\begin{aligned} W(\vec{k}) &= \sum_{m=0}^{M-1} w_m e^{j\vec{k}\vec{x}} \\ W(k_x, k_y, k_z) &= \sum_{m=0}^{M-1} w_m e^{j(k_x x_m + k_y y_m + k_z z_m)} \\ W(k_x, k_y, k_z) &= \sum_{m=0}^{M-1} w_m e^{jk(\sin\theta\cos\phi x_m + \sin\theta\sin\phi y_m + \cos\theta z_m)} \\ W(k_x, k_y, k_z) &= \sum_{m=0}^{M-1} w_m e^{j\frac{2\pi f}{c}(\sin\theta\cos\phi x_m + \sin\theta\sin\phi y_m + \cos\theta z_m)} \end{aligned} \quad (6)$$

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