Array gain and reduction of self-noise

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***Abstract*—Because an array is composed of many sensors, each making an independent measurement of the wavefield, intelligent combination of the sensor signals should lead to an overall increase in the signal-to-noise ratio. What we are trying to achieve is enhancing the signal with an array beyond what is possible with a single sensor. The array gain measures an array's signal-to-noise ratio enhancement, and thus also the self-noise improvement of the total array.**

***Index Terms*—Array gain, self-noise, signal-to-noise ratio, delay-and-sum beamforming**

I. Delay-and-sum beamforming

One of the most common and robust beamforming algorithms is the conventional beamformer, also known as delay-and-sum (DAS) beamforming. Consider an array consisting of M sensors located at position $\vec{x}_m$ and measures a wavefront $s(\vec{x}, t)$. The waveform spatially sampled by the mth sensor is $y_m(t) = s(\vec{x}_m, t)$. The DAS beamformer consists of applying a delay Δ_m and an amplitude weight w_m to the output of each sensor, and then summing the resulting signals.

$$z(t) = \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m) \tag{1}$$

The delays are chosen to maximize the array's sensitivity to waves propagating from a particular direction. By adjusting the delays, the array's direction of look can be steered towards the source, and the waveforms captured by the individual sensors add constructively. The expression in (1) is then reduced to

$$z(t) = s(t) \cdot \left[\sum_{m=0}^{M-1} w_m \right] \tag{2}$$

and the output of the array equals the waveform radiated by the source times a constant.

II. Signal-to-noise ratio

A. Single sensor

Consider a single sensor receiving the wavefront $s(t)$ as

$$y(t) = s(t) + n(t) \tag{3}$$

where $n(t)$ represents the noise which may be due to thermal noise in the electronics, or noise from the environment. The signal-to-noise ratio (SNR) is defined as the signal power over the noise power. Assuming the signal and noise are uncorrelated, the SNR for a single sensor is given as

$$SNR_{sensor} = \frac{E\left\{|s(t)^2|\right\}}{E\left\{|n(t)^2|\right\}} = \frac{\sigma_s^2}{\sigma_n^2} \tag{4}$$

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B. Array

Now consider an array of M elements rather than a single sensor. The signal received by the sensor m is

$$y_m(t) = s_m(t) + n_m(t) \tag{5}$$

Using the DAS beamformer as given in (1), the output may be stated as

$$\begin{aligned} z(t) &= \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m) \\ &= \sum_{m=0}^{M-1} w_m s_m(t - \Delta_m) + n_m(t - \Delta_m) \\ &= z_s(t) + z_n(t) \end{aligned} \tag{6}$$

Now assuming the delays are chosen to match the wave's direction of propagation, and that the noise is spatially uncorrelated from sensor to sensor, we can calculate the signal power and the noise power of the entire array of M sensors. Omitting the summation variables of the M sensors for notational convenience, the signal power of the array is given as

$$\begin{aligned} E\{|z_s(t)^2|\} &= E\{|\sum_m w_m s_m(t)|^2\} \\ &= E\{\sum_m w_m s(t) \cdot \sum_l w_l^* s_l^*(t)\} \\ &= E\{|\sum_m w_m|^2\} \cdot E\{|s(t)^2|\} \\ &= |\sum_m w_m|^2 \cdot \sigma_s^2 \end{aligned} \tag{7}$$

Likewise, the signal power of the noise of the array may be calculated as

$$\begin{aligned} E\{|z_n(t)^2|\} &= E\{|\sum_m w_m n_m(t)|^2\} \\ &= E\{\sum_m w_m n_m(t) \cdot \sum_l w_l^* n_l^*(t)\} \\ &= E\{\sum_m \sum_l w_m w_l^* R_n(m-l)\} \end{aligned} \tag{8}$$

where R_n denotes the correlation function of the noise. If we assume the noise is spatially uncorrelated from sensor to sensor, we can write the correlation function as

$$R_n(m-l) = \left\{ \begin{array}{ll} \sigma_n^2 & \text{for } m = l \\ 0 & \text{otherwise} \end{array} \right\}$$

so that the total noise power of the array is given as

$$E\{|z_n(t)^2|\} = \sum_m |w_m|^2 \cdot \sigma_n^2 \tag{9}$$

Comparing the array signal power in (7), and the array noise power in (9), we see that the signal power is multiplied by a factor $|\sum_m w_m|^2$, whereas the noise power is multiplied by $\sum_m |w_m|^2$. For our simplified case where the delays match the incoming wave, and the noise is spatially uncorrelated at the sensor locations, the total array SNR is given as

$$SNR_{array} = \frac{|\sum\limits_m w_m|^2 \cdot \sigma_s^2}{\sum\limits_m |w_m|^2 \cdot \sigma_n^2} \tag{10}$$

III. Array gain

The array gain is defined as the ratio of the array SNR and the individual sensor SNR, and measures the enhancement when using M sensors rather than a single sensor.

$$G = \frac{SNR_{array}}{SNR_{sensor}} \tag{11}$$

Using the expressions for the sensor SNR given in (4) and the array SNR in (10), the total array gain is given as

$$G = \frac{\left|\sum\limits_{m=0}^{M-1} w_m\right|^2}{\sum\limits_{m=0}^{M-1} |w_m|^2} \tag{12}$$

which means that the array gain only depends on the weighting w applied to the sensors. Considering the case with uniform weighting $w_m = w$, the array gain is simply $G = (M^2|w|^2)/(M|w|^2)$ which equals M *independent* of the element positions and the sensor weights. Thus we can increase the signal-to-noise ratio by a factor of M by using an array rather than a single sensor.

$$SNR_{array} = 10log(M) + SNR_{sensor} \tag{13}$$

IV. Reduction of wind noise by using an array

To improve the SNR the array must add signal coherently, and noise incoherently. Wind noise can be viewed as spatially white so that wind noise sampled at different places in space is not correlated from position to position. When in the beamforming algorithm many different signals are added from many microphones, the wind noise (because it is spatially white) will be added out of phase and attenuated proportional with the number of microphones being used. The attenuation is given by the array gain of the array.

V. Self noise of Nor848A acoustic cameras

The sensors being used in the Nor848A acoustic camera have a self noise of 33 dB and an A-weighted SNR of 61 dB. For an array of sensors, the self-noise of the array is reduced, and the SNR is increased, by $10log(M)$ where M is the number of microphones bein used.

Type	Self-noise	SNR
Nor848A-4	12 dB	82 dB
Nor848A-10	9 dB	85 dB
Nor848A-16	7 dB	87 dB

References

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